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UNSTEADY NONSIMILAR NATURAL CONVECTION OVER A VERTICAL FLAT PLATE IN A THERMALLY STRATIFIED FLUID

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NOMENCLATURE

a	ambient temperature gradient, dT_∞/dx
C_f	local skin-friction coefficient
F, F_w	dimensionless stream function and mass-transfer parameter, respectively
F''_w, G'_w	surface skin-friction and heat-transfer parameters, respectively
g, G	gravitational acceleration and dimensionless temperature, respectively
Gr_x, k	local Grashof number and thermal conductivity, respectively
Nu, Pr	local Nusselt number and Prandtl number, respectively
q	local heat-transfer rate per unit area
t, t^*	dimensional and dimensionless times, respectively
$T, T_\infty, T_{\infty 0}$	temperature, ambient temperature, and ambient temperature at $x = 0$, respectively
T_w, T_{w0}	wall temperature and its value at $t = 0$, respectively
x, y	distances along and perpendicular to the surface, respectively.

Greek symbols

β	bulk coefficient of thermal expansion
ε	constant
η, ξ	transformed coordinates
ν, ρ	kinematic viscosity and density, respectively
τ, ϕ, ψ	shear stress at the surface, function of t, and dimensional stream function, respectively.

Superscript

$'$	differentiation with respect to η.

Subscripts

w, ξ	condition at the surface and derivative with respect to ξ, respectively.

INTRODUCTION

THE AMBIENT fluid in many natural or mixed convection flows both in technology and in nature is stably stratified. Thermal stratification is commonly encountered in the atmosphere and in lakes and cooling ponds. Also heat rejection from power plants and other industrial systems often involves both natural and mixed convection flows in stratified media. The steady free convection flow over a vertical flat plate in a stably stratified medium has been studied by several authors [1–4]. However, the analogous unsteady problem has not been studied so far.

The aim of this note is to study the unsteady incompressible laminar free convection boundary layer for nonsimilar flow over a vertical flat plate in a stratified medium when the wall temperature varies with time. The partial differential equations, with three independent variables governing the flow, have been solved numerically using an implicit finite-difference scheme in combination with the quasilinearization technique [5, 6]. The results have been compared with those available in the literature.

GOVERNING EQUATIONS

We consider a vertical plate immersed in a stable thermally stratified fluid. The wall temperature is assumed to vary continuously with time and the ambient temperature with distance from the leading edge. The boundary layer equations for the unsteady laminar incompressible natural convection flow on a vertical flat plate with mass transfer using the Boussinesq approximations for the density variation and

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assuming the other properties to be constant can be expressed as [1–4]

$$F''' + 3FF'' - 2F'^2 + G - F_{t^*} = 4\xi(F'F'_\xi - F''F_\xi), \quad \text{(1a)}$$

$$Pr^{-1}G'' + 3FG' - 4\xi F' - G_{t^*} = 4\xi(F'G_\xi - G'F_\xi). \quad \text{(1b)}$$

The boundary conditions are given by

$$\left.\begin{aligned} &F(\xi,0,t^*) = F_w, \quad F'(\xi,0,t^*) = G(\xi,0,t^*) - 1 - \phi(t^*) + \xi = 0, \\ &F'(\xi,\infty,t^*) = G(\xi,\infty,t^*) = 0. \end{aligned}\right\} \quad (2)$$

The initial conditions are given by steady-state equations which are obtained from equations (1a) and (1b) by putting $t^* = F_{t^*} = G_{t^*} = 0$ in them and they are [1–4]

$$F''' + 3FF'' - 2F'^2 + G = 4\xi(F'F'_\xi - F''F_\xi), \quad \text{(3a)}$$

$$Pr^{-1}G'' + 3FG' - 4\xi F' = 4\xi(F'G_\xi - G'F_\xi), \quad \text{(3b)}$$

where

$$\left.\begin{aligned} &\xi = ax/\Delta T_{w0}, \quad \eta = [g\beta\Delta T_{w0}(4xv^2)^{-1}]^{1/4}y, \\ &t^* = [(g\beta\Delta T_{w0})/(2x)]^{1/2}t, \quad \Delta T_{w0} = T_{w0} - T_{\infty 0}, \end{aligned}\right\} \quad \text{(4a)}$$

$$\psi(x,y,t) = 4^{3/4}v^{1/2}(g\beta\Delta T_{w0})^{1/4}x^{3/4}F(\xi,\eta,t^*),$$

$$u = \partial\psi/\partial y, \quad v = -\partial\psi/\partial x, \quad G(\xi,\eta,t^*) = (T - T_\infty)/\Delta T_{w0}, \quad \text{(4b)}$$

$$\left.\begin{aligned} &T_\infty = T_{\infty 0} + ax, \quad a = \mathrm{d}T_\infty/\mathrm{d}x > 0, \\ &\qquad (T_w - T_{w0})/\Delta T_{w0} = \phi(t^*), \\ &F_w = -(4\xi)^{-3/4}Gr_x^{-1/4}\int_0^\xi [v_w/(v/x)]\xi^{-1/4}\,\mathrm{d}\xi, \\ &Gr_x = g\beta\Delta T_{w0}x^3v^{-2}. \end{aligned}\right\} \quad \text{(4c)}$$

If the normal velocity at the wall $v_w/(v/x)$ is taken as a constant, then F_w will be a constant ($F_w > 0$ for suction and $F_w < 0$ for injection). It may be remarked that the steady-state equations (3a) and (3b) are the same as those of refs. [1–4].

The skin-friction coefficient and heat-transfer coefficient (Nusselt number) can be written in the form [3, 4]

$$\left.\begin{aligned} C_f &= \tau_w/[\rho(v/x)^2] = 4(Gr_x/4)^{3/4}F''_w, \\ Nu &= qx/k\Delta T_{w0} = -(Gr_x/4)^{1/4}G'_w. \end{aligned}\right\} \quad (5)$$

RESULTS AND DISCUSSION

Equations (1a) and (1b) have been solved numerically under boundary conditions (2) and initial conditions (3a) and (3b) using an implicit finite-difference scheme in combination with the quasilinearization technique. Since the method is described in detail in refs. [5, 6], its description is omitted here. In order that the solution of the difference equations converges to the true solution, the effect of step sizes $\Delta\eta$, $\Delta\xi$, and Δt^* on the solution has been studied and their optimum values have been obtained. Consequently, we have taken $\Delta\eta = 0.05$, $\Delta\xi = 0.1$, and $\Delta t^* = 0.1$ for computation. In addition, we have taken the value of the edge of the boundary layer (η_∞) between 5 and 20 depending on the values of Pr and F_w. Further reduction in step size and η_∞ changes the results only in the fourth decimal place. For computation the difference between the wall and ambient temperatures is taken as $\phi(t^*) = \varepsilon t^*$.

In order to test the accuracy of our method, we have compared our steady-state ($t^* = 0$) results for $\xi = 0$ and $F_w = 0$ with those of ref. [1] and they agree at least up to the fourth decimal place. Hence the comparison is not shown here. As a further test for the accuracy of our method, we have compared our steady nonsimilar flow results without mass transfer ($F_w = 0$) with those of Venkatachala and Nath [4] who used an implicit finite-difference scheme. We have also compared our results with those of the series solution method [1] and the local nonsimilarity method [3]. This has been done to assess the accuracy of these approximate methods which have widely been used in the literature. Since the nature of the variation of the skin-friction results is the same as the heat-transfer results, for the sake of brevity, the comparison of only heat-transfer results is shown in Fig. 1. The results are found to be in

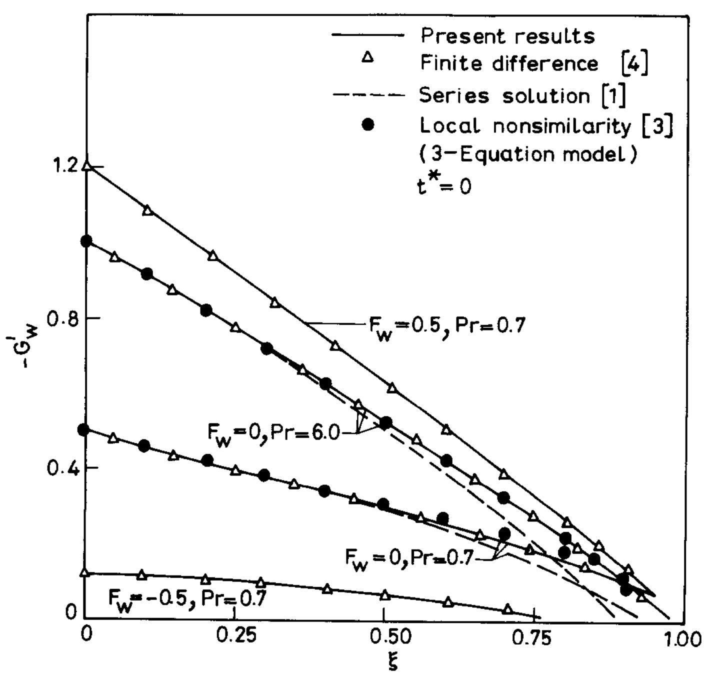

FIG. 1. Comparison of heat-transfer results for the steady-state case.

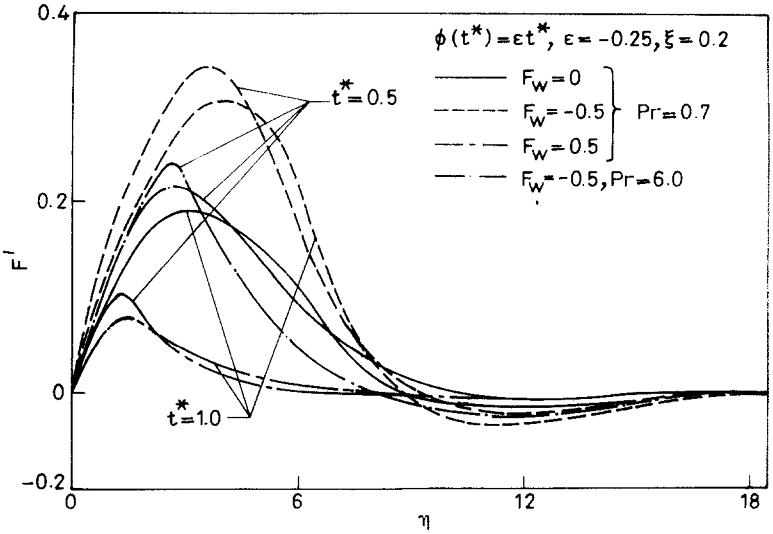

FIG. 2. Velocity profiles.

excellent agreement with those of ref. [4]. As expected the local nonsimilarity and series solution results differ from our results for large ξ and small Pr. The detailed discussion on the predictive capabilities of these approximate methods is given in ref. [4]. In the subsequent paragraphs, we present results for $t^* > 0$.

The velocity and temperature profiles (F', G) for $t^* = -0.5$, 1.0, $\xi = 0.2$, $F_w = 0.5, 0, 0.5$, and $Pr = 0.7, 6$ are presented in Figs. 2 and 3, respectively. It is observed that undershoot arises both in the velocity and temperature profiles in the outer region of the boundary layer for the case of injection or no mass transfer. The magnitude of the undershoot increases with time t^*, since we have assumed that the difference between the wall temperature and the ambient temperature decreases with time. If this difference increases with time, then the opposite effect is observed (not shown in Figs. 2 and 3). On the other hand, suction tends to prevent or reduce the undershoot. Physically, the temperature undershoot (inversion) arises because at any given location ξ, the fluid coming from below in the outer region of the boundary layer is colder than the ambient fluid at that height. If there is a rapid increase in the ambient temperature, the colder fluid is not able to attain the ambient temperature at this level which causes undershoot in the temperature. Due to the temperature undershoot the thermal boundary layer becomes thicker. This temperature defect gives rise to reverse flow in the outer region of the boundary layer though the main flow is largely undisturbed. For small Pr ($Pr = 0.7$), reversal of flow occurs for large η, whereas for large Pr, it occurs for small values of η. A similar trend has been observed by Eichhorn [1] and Fujii *et al.* [2] for the steady-state case without mass transfer.

The variation of skin-friction and heat-transfer parameters $(F''_w, -G'_w)$ with time t^* for $Pr = 0.7$ and 6 and $\xi = 0.2$ is shown in Fig. 4. Since we have taken the difference between the wall

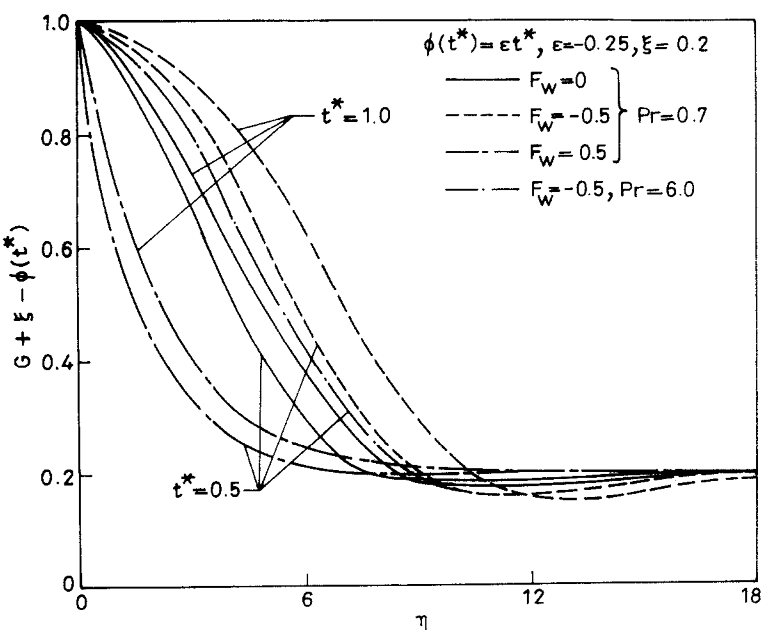

FIG. 3. Temperature profiles.

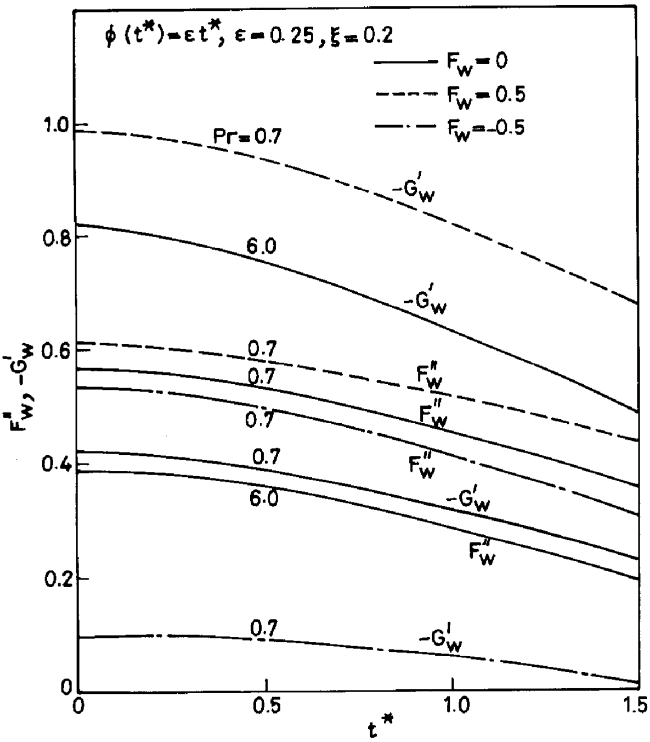

FIG. 4. Skin-friction and heat-transfer results.

temperature T_w and the ambient temperature $T_{\infty 0}$ to decrease with time, both skin friction and heat transfer decrease as time increases whatever the values of Pr and ξ. The opposite holds good if $(T_w - T_{\infty 0})$ increases with time t^* (not shown in Fig. 4). However, the effect is more pronounced on the heat transfer. It is found that the heat-transfer parameter G'_w is increased due to the increase in Prandtl number because the thermal boundary layer decreases with Pr which results in a higher temperature gradient at the wall and hence a higher heat transfer. On the other hand, the skin friction (F''_w) is reduced as Pr increases because higher values of Pr increase the momentum boundary layer thickness which reduces the skin friction. Also injection reduces both the skin friction and heat transfer. This is due to the thickening of both momentum and thermal boundary layer thicknesses. Furthermore, both skin friction and heat transfer $(F''_w, -G'_w)$ decrease as ξ increases. A similar effect has been observed by previous investigators [1–4] for the steady-state case. Here we have not shown the effect of the variation of ξ on F''_w and $-G'_w$ for $t^* > 0$ as the trend is similar to that of the steady-state case ($t = 0$) which has already been studied in detail [1–4].

CONCLUSIONS

The heat transfer is strongly affected by the variation of the wall temperature with time and Prandtl number as compared to skin friction. Both skin friction and heat transfer decrease or increase with time if the difference between the wall temperature and ambient temperature decreases or increases with time. They also decrease with streamwise distance and injection. The effect of suction is just the reverse. Beyond a certain critical streamwise location, the velocity and temperature profiles show reversal of flow and temperature inversion, respectively, and their magnitude increases with time and injection. The effect of suction is just the reverse.

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